Analytical Modeling of Electrostatic Membrane Actuator for Micro Pumps

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Abstract-A large class of micro pumps consists of a cavity in a substrate, a diaphragm that seals the cavity, and inlets and outlets for the cavity which are controlled by valves. The cavity is initially filled with a fluid. The diaphragm is then actuated by a voltage between the diaphragm and the cavity floor to compress the fluid. When the pressure exceeds a certain value, the fluid is expelled. During actuation, the electrostatic attractive force of the substrate and the pressure rise in the fluid lead to the bending and stretching of the diaphragm. Thus, the prediction of the pump performance (e.g., fluid pressure, diaphragm stresses) requires the solution of a coupled nonlinear elasto-electro-hydrodynamic problem. In this paper, a simplified analytical model is developed to predict the state of an electrostatically actuated micro pump at equilibrium. The state includes the deformed shape and the internal stresses of the diaphragm and the pressure of the fluid when the actuator is subjected to a given applied dc voltage. The model is based on the minimization of the total energy consisting of the capacitive energy, the strain energy of the diaphragm, and the energy of the fluid which is considered to be an ideal gas. The method is employed to study two pumps, one with an axisymmetric single cavity, and the other with an axisymmetric annular cavity (a cavity with an island in the middle). In the former case, upon actuation, the diaphragm contacts the cavity floor from the outer periphery. Thus, energy is a function of the radius of the contact front, and equilibrium configuration is achieved at a radius where the derivative of the energy with respect to the radius vanishes. In the latter case, upon actuation, the diaphragm contacts the cavity from both of the inner and the outer peripheries. Here, equilibrium is reached when the derivatives of the energy with respect to the radii of both of the inner and outer periphery contact points vanish. It is expected that most practical pumps can be analyzed by one of the two formulations presented in the paper. Our analyses of both pumps indicate that the pressure of the gas at equilibrium increases only slightly when the stiffness of the diaphragm is increased, whereas it changes nearly inversely with the thickness of the dielectric between the diaphragm electrode and the cavity floor. Also, as expected, the pressure increases as the initial volume of the cavity (i.e., the volume of the gas to be compressed) is decreased. Furthermore, we find that the calculated stresses in the diaphragm do not exceed the typical yield stress values of many glassy polymers, a candidate material for the diaphragm. Therefore, the assumption of a linear elastic diaphragm employed in the proposed model does not put a limitation on the predictions. Dielectric breakdown may be a limiting factor for the maximum attainable pressure rather than the mechanical strength of the diaphragm material. Although stresses are low, they may be severe enough to cause delamination between different layers in the diaphragm, too. [402]

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I. INTRODUCTION

RECENT development in micro fluidics has resulted in a variety of small-scale pumps and compressors [1]–[3]. These devices have a wide range of potential applications, such as drug delivery [4], localized cooling of electronic circuits, and inkjet printing. Typically, a micro pump or a compressor consists of a thin film membrane that seals a cavity with inlet and outlet ports controlled by valves. The membrane can be actuated by electrostatic [5], piezoelectric [6], and thermal [7] means to fill the cavity with a fluid (liquid or gas) and to pressurize it. As the pressure exceeds a certain limit, the liquid is pumped out.

As in the case of any engineering component, simulation and modeling of micro pumps are essential for better understanding of their functionality and optimum design. The simulation, however, is nontrivial because, in general, it involves coupled electrostatic, elastodynamic, thermal, and fluid dynamic interactions [8]. The problem is nonlinear as well. For example, the forcing on the pump membrane depends nonlinearly on the deformation of the membrane itself. The problem is further complicated by the contact between the membrane and the cavity substrate during operation. Additional nonlinearities may be contributed by the nonlinear constitutive behavior of the materials involved. In this work we consider elastic strain energy, capacitive energy, and pressure work contributions.

Robust numerical methods, such as finite element analysis, may be employed for the simulation of micro pumps [9], [10]. Such analysis, however, is prohibitively time consuming for parametric studies. Hence, analytical modeling of micro pumps, based on reasonable approximations, is desirable to study the dependence of pump performance on various parameters.

In this paper, an analytical model is developed for quasistatic analysis of a pump/compressor designed for a refrigeration system. The pump consists of a cavity filled with a gas (refrigerant) and a membrane. It is actuated by applying a known voltage between the membrane and the cavity. The membrane is thus pulled toward the cavity, and the gas is compressed. The model predicts the pressure of the gas and the shape of the membrane at equilibrium for a given applied voltage between the cavity and the membrane. The model is employed to analyze two basic axisymmetric micro pumps—one with a simple cavity (like a bowl), the other with

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Fig. 1. Schematic of pump 1.



Fig. 2. Schematic of pump 2.

a doughnut-shaped cavity. The essential difference between the pumps lies in the boundary conditions of their membranes. It is expected that the analysis of most practical pumps can be carried out using the analysis of these two basic pumps.

II. ELECTROSTATICALLY ACTUATED MICRO PUMP/COMPRESSOR

Figs. 1 and 2 show the two electrostatically actuated micro pumps to be considered in this paper. They will be referred to as pump 1 and pump 2, respectively. Each consists of an axisymmetric cavity and a thin film membrane that seals the cavity. The cavity has inlet and outlet channels controlled by valves for the transport of gases. Initially, the chamber is filled with gas and the valves are closed. The pump is then actuated by applying a voltage between the membrane and the substrate. For pump 1, the diaphragm contacts the floor of the cavity from the outside perimeter (Fig. 1) and the contact front advances toward the center. Thus, the gas inside the cavity is compressed until the diaphragm reaches a steady state equilibrium for a given applied voltage. For pump 2, the diaphragm contacts the substrate at both the outside and the inside perimeters after actuation, and the two contact fronts advance toward each other until equilibrium is reached (Fig. 2). The voltage between the membrane and the cavity may be different in the two contact regions. The membrane of the actuator may consist of multiple layers of metal and dielectric. It is rigidly bonded to the substrate on the outside perimeter. The cavity may also be coated by a thin layer of dielectric. The total

thickness of the dielectric layer between the metal electrode of the membrane and the substrate is d. The thickness of the metal electrode of the membrane is expected to be much smaller than the total thickness of the membrane, t. Hence the metal layer will not provide any structural stiffness. The interested reader is directed to [11] for the fabrication issues of the silicon compression chamber.

The central theme of this paper is the formulation of a method for seeking the equilibrium configuration of the micro pump when actuated by a given voltage (time independent), and hence to quantify the pressure, diaphragm shape, and its stresses and strains at equilibrium.

III. ANALYTICAL MODEL OF THE ELECTROSTATIC ACTUATION OF MICRO PUMP

The model is based on several simplifying assumptions.

A. Assumptions

- The membrane material is isotropic and behaves linear elastically.
- There is no residual or intrinsic stress [12] in the membrane. If there is a known residual stress, it can easily be incorporated in the model. Thin films are usually subjected to residual stress, but various methods have been developed to relieve the stress, such as annealing [12].
- The thickness of the membrane is several orders of magnitude smaller than the lateral dimension (diameter). Thus, bending stresses are ignored. Only in-plane stresses are considered, and they are uniform across the thickness.
- Tangential strain of the membrane is considered negligible ($\epsilon_{\theta} = 0$), and hence the fixed boundary condition along the outer perimeter of the membrane is satisfied. A more general analysis with $\epsilon_{\theta} \neq 0$ due to Hencky [13] will be carried out for pump 1. The analysis will justify the " $\epsilon_{\theta} = 0$ " assumption.
- The slopes of the deformed membrane and the cavity with respect to the radial direction are much smaller than unity.
- The membrane-substrate contact is frictionless. It is expected that a thin hydrodynamic layer of fluid between the film and the substrate will be present during compression that will reduce the friction significantly.
- The compression of the gas satisfies the ideal gas law at constant temperature: Pv = constant, where P is the pressure and v is the volume, which implies that the temperature of the gas does not change. Since the number of gas molecules in the compression chamber remains constant, the internal energy of the gas also remains constant. Thus, the work done on the gas during compression is converted to heat and must be released to the environment through the diaphragm and the cavity substrate. The constitutive behavior Pv = constant is chosen to demonstrate the applicability of the method developed in this paper. Other constitutive behavior, such as that which involves phase transformation of the gas, can also be incorporated.

B. The Model

During actuation, work is done on the membrane and the gas. Thus, both the strain energy of the membrane, U_s , and the work done on the gas, $U_p = -\int P \, dv$, increase. However, as the membrane contacts the substrate and the contact front advances, electrical (capacitive) work is done by the capacitor formed by the membrane and the cavity floor, since opposite charges come closer toward each other. This work is the capacitive energy, U_c , of the capacitor, which also increases during actuation. The equilibrium configuration of the pump after actuation with a given applied voltage is obtained from the first principle of energy conservation: a small virtual perturbation from the equilibrium configuration will cause small changes in the strain energy of the membrane, dU_s , work done on the compressed gas, dU_p , and the capacitive energy, $-dU_c$. Energy conservation dictates that the net change of energy is zero, i.e., $dU = d(U_s + U_p - U_c) = 0$. Note that a decrease in the electrostatic potential energy U_c $(=\frac{1}{2}CV^2)$, where C is the capacitance and V is the voltage) due to opposite charges coming closer to each other causes an increase in U_s and U_p . Hence the negative sign with dU_c .

This change of configuration is achieved, in case of pump 1, by a change of radius R_1 of the contact front between the membrane and the cavity (Fig. 1). In case of pump 2, the change can be induced independently by changes in the inner and the outer contact front radii, r_1 and r_2 , respectively (Fig. 2). Thus, the equilibrium configuration is defined by $(dU/dR_1) = 0$ for pump 1 and $(\partial U/\partial r_1) = (\partial U/\partial r_2) = 0$ for pump 2. Here, R_1 , r_1 , and r_2 can be viewed as the generalized coordinates of the problem. Note that the energy derivative and not the total energy is employed to solve for the equilibrium configuration.

 dU_s and dU_p are computed from the derived closed-form expressions of U_s and U_p . Computation of dU_c is nontrivial since it involves 1) the capacitive energy $U_{||}$ between the diaphragm and the cavity floor where they are in contact, and 2) the capacitive energy U_{cav} of the cavity (diaphragm and the cavity floor are not in contact). Thus $U_c = U_{||} + U_{cav}$. Evaluation of U_{cav} poses difficulty. However, it is shown in the following that the energy derivative $|(dU_c/dR)| \approx$ $|(dU_{||}/dR)|$.

Consider the capacitor formed by the two axisymmetric electrodes in Fig. 3. The radius of the cavity is R, its middle gap is D, and the gap between the parallel plates is d, such that $D \gg d$. The dielectric constants of the cavity and the parallel plate regions are $\kappa_{\rm cav}$ and $\kappa_{\rm ||}$, respectively, where $\kappa_{\rm cav} \leq \kappa_{\rm ||}$. The electrodes forming the cavity meet each other tangentially along the perimeter of the cavity, and their geometry near the perimeter does not change with a variation in R. Let $U_{\rm ||}$ and $U_{\rm cav}$ be the capacitive energies between the parallel electrodes and the electrodes forming the cavity due to an applied voltage V between them. Then $|(dU_{\rm ||}/dR)| \gg |(dU_{\rm cav}/dR)|$. Hence $|(dU_c/dR)| \approx |(dU_{\rm ||}/dR)|$, where $U_c = U_{\rm ||} + U_{\rm cav}$.

Proof: The change of capacitance between the parallel plates, dC_{\parallel} , due to a change in R, dR, is

$$\frac{dC_{||}}{dR} = \frac{2\pi R}{d} \epsilon_0 \kappa_{||} \tag{1}$$



Fig. 3. Electrodes with cavity and parallel-plate regions.



Fig. 4. Comparison of parallel-plate and total capacitance.

where ϵ_0 is the permittivity constant. Let C_{cav} be the total capacitance of the cavity, most of which is contributed by the region near the perimeter of the cavity. Let $C_0 = C_{\text{cav}}/(2\pi R) = \text{capacitance/unit length of the cavity perimeter.}$ If the cavity shape near the perimeter does not change with R, or changes slightly with R, then $dC_0/dR \approx 0$. Thus

$$\frac{dC_{\text{cav}}}{dR} = \frac{d}{dR}(2\pi RC_0) = 2\pi C_0.$$
 (2)

In order to compare the right-hand sides of (1) and (2), we see that $(R \times 1)\epsilon_0\kappa_{||}/d$ is the capacitance between two parallel electrodes of length R, of width unity, and with a gap of d between them. C_0 , by its definition, is the capacitance between two curved, pie-shaped electrodes of length R, but of a width varying from unity to zero, and with a gap varying from d to D as shown in Fig. 4. Clearly, $C_0 \ll \frac{R}{d}\epsilon_0\kappa_{||}$ when $D \gg d$. Thus from (1) and (2), $|(dC_{||}/dR)| \gg |(dC_{cav}/dR)|$. Since the capacitive energy is given by $CV^2/2$, where V is the applied voltage between the capacitor electrodes, $|(dU_{||}/dR)| \gg |(dU_{cav}/dR)|$, and hence

$$\frac{dU_c}{dR} \approx \frac{dU_{||}}{dR}.$$
(3)

For the case of the micro pumps under consideration, the contact region of the membrane and the cavity forms the parallel plate capacitor C_{\parallel} , whereas the rest of the membrane and the cavity represent the cavity of Fig. 3. Due to this similarity, the corresponding energy derivatives dU_{\parallel}/dR and dU_{\parallel}/dr_i , i = 1, 2 are used as the derivatives of the total capacitive energy.

The deformation of the membranes of pumps 1 and 2 are different due to their boundary conditions. Hence, for convenience, they will be treated in separate sections in the following.

IV. ANALYSIS OF PUMP 1-SINGLE CAVITY

Fig. 1 shows the pump configuration at equilibrium after actuation by a voltage V applied between the membrane and the cavity. The equilibrium is defined by the radius R_1 of the contact front between the diaphragm and the cavity. The shape of the cavity is given by $z_0(r)$, with $z_0(0) = 0$ at the bottom of the cavity (the origin for cavity). The diaphragm is defined by $z_f(r)$ in $r \sim (0, R_1)$ with $z_f(0) = 0$ at the top of the membrane (the origin for the membrane). Note that z_0 is an arbitrarily prescribed function with $|z'_0| \ll 1$. The shape of the membrane z_f is derived next.

A. Membrane Deformation Under Uniform Pressure

Consider a thin circular membrane of radius R_1 , clamped along the perimeter, being subjected to a net pressure p. Let Sbe the tension per unit length of the membrane. A section of the membrane of radius r is considered to evaluate the forces on it. The net force on the membrane perpendicular to the section due to p is $\pi r^2 p$. The restoring force due to tension in the membrane is $2\pi r S z'_f$, where the prime denotes derivative with respect to r. Force equilibrium requires

$$2\pi r S z' + p\pi r^2 = 0. (4)$$

The solution is given by

$$z_f = -\frac{p}{4S}r^2\tag{5}$$

which satisfies the $z_f(r = 0) = 0$ condition. In order to evaluate S, linear elastic property of the membrane is employed. Strain along the radial direction ϵ_r is

$$\epsilon_r = \left(\int_0^{R_1} \sqrt{1 + z_f'^2} \, dr - R_1 \right) / R_1$$
$$\approx \int_0^{R_1} \left(z_f'^2 / 2 \right) dr = \frac{p^2 R_1^2}{24S^2}. \tag{6}$$

Also, $\epsilon_{\theta} = 0$ implies that the tangential and radial stresses are related by $\sigma_{\theta} = \nu \sigma_r$, where ν is the Poisson's ratio of the diaphragm material. Thus $\epsilon_r = \sigma_r/E_*$, where $E_* = E/(1-\nu^2)$. By definition, $S = \sigma_r t = \epsilon_r E_* t$, which, together with (6), gives

$$S = \left[\frac{1}{24}E_*tp^2R_1^2\right]^{(1/3)}.$$
(7)

Thus, for a given p, z_f is completely known as a function of diaphragm radius R_1 [(5) and (7)].

B. Strain Energy U_s of the Membrane

Fig. 1 shows the equilibrium configuration of the membrane after actuation with voltage V. The membrane is subjected to a net pressure p. The strain energy of the film, U_s , is the integral of the energy density over the volume of the diaphragm

$$U_s = \pi t \int_0^R \epsilon_r \sigma_r r \, dr = \pi t \epsilon_r^2 E_* R^2 / 2 \tag{8}$$

where $\sigma_r = E_* \epsilon_r$ is used, and the strain, ϵ_r , is due to the deformation of the film as shown in Fig. 1. Thus

$$\epsilon_r = \left[\int_0^{R_1} \sqrt{1 + z_f'^2} \, dr + \int_{R_1}^R \sqrt{1 + z_0'^2} \, dr - R \right] \middle/ R \tag{9}$$

where z_f is given by (5) and (7).

If the film has a residual stress, then the strain energy U_s of the diaphragm in (8) needs to be modified to

$$U_s = \pi t \int_0^R \epsilon_r (\sigma_0 + \sigma_r) r \, dr \tag{10}$$

where $\sigma_r = E_* \epsilon_r$, and σ_0 is the known biaxial residual stress.

C. Parallel Plate Capacitive Energy $U_{||}$ Between the Membrane and the Cavity

$$U_{||}$$
 is given by

$$U_{\parallel} = \frac{1}{2}CV^2$$
, where $C = \epsilon_0 \kappa \frac{A}{d}$

where A is the contact area between the membrane and the cavity, and κ is the dielectric constant of the dielectric between the cavity floor and the membrane electrode. Thus

$$A = \int_{R_1}^{R} 2\pi r \sqrt{1 + z_0'^2} \, dr \approx 2\pi \int_{R_1}^{R} r \left(1 + \frac{1}{2} z_0'^2\right) dr$$

and

$$U_{||} = \epsilon_0 \frac{\kappa}{d} \pi \left[\int_{R_1}^R r \left(1 + \frac{1}{2} z_0^{\prime 2} \right) dr \right] V^2.$$
(11)

Here, the slopes of the diaphragm and the cavity z_i (i = 0 for cavity, =f for diaphragm) are small so that $(dz_i/dr)^2 \ll 1$, and the integrand $\sqrt{1+z_i'^2}$ is approximated by $1+z_i'^2/2$. If in case $z_i'^2$ is not small, then the integrations must be carried out numerically with $\sqrt{1+z_i'^2}$.

D. Pressure Energy U_p

Before actuation, diaphragm is horizontal. Initial pressure and volume within the cavity are p_0 and v_0 , respectively. The pressure outside the cavity is p_0 . During actuation, gas in the cavity is compressed as shown in Fig. 1 and its pressure increases from $P = p_0$ to $P = p_0 + p$, and its volume decreases from v_0 to v. The pressure outside the cavity remains constant at p_0 . The gas satisfies the constitutive behavior $Pv = p_0v_0 = C_* = \text{constant}$. Thus, the work done *on* the gas inside the cavity is

$$U_{p-\text{inside}} = -\int_{v_0}^{v} P \, dv \tag{12}$$

where the initial (before actuation, membrane horizontal) and the final volumes of the cavity, v_0 and v, are given by

$$v_0 = \int_0^R \pi r^2 \, dz_0 \tag{13}$$

$$v = \int_0^{R_1} \pi r^2 (dz_0 + dz_f).$$
(14)



Fig. 5. Variation of total energy U as a function of R_1 for pump 1. System is in equilibrium, when $dU/dR_1 = 0$, which occurs at $R_1 = 3.798$ mm.

The change of volume outside the cavity is also $(v_0 - v)$. Thus, the work done by the gas outside the cavity is

$$U_{p-\text{outside}} = p_0(v_0 - v).$$
 (15)

The total work done *on* the gas inside and outside the cavity is

$$U_{p} = U_{p-\text{inside}} - U_{p-\text{outside}}$$

= $-\int_{v_{0}}^{v} P \, dv - p_{0}(v_{0} - v)$
= $-C_{*} \ln(v/v_{0}) - p_{0}(v_{0} - v).$ (16)

E. Equilibrium Configuration

Note that U_s , U_p , and $U_{||}$ are functions of p and R_1 . However, p can be solved from $(p_0 + p)v = C_*$

$$p = \frac{C_*}{v} - p_0$$
 (17)

where v is defined as a function of p and R_1 in (14). Thus, the energies are represented as explicit functions of R_1 .

 R_1 is solved from the condition of equilibrium, i.e., $(dU/dR_1) = (d(U_s + U_p - U_{||})/dR_1) = 0$. All three components of energy, and their derivatives, are evaluated symbolically by MATHEMATICA¹ using (8), (11), and (16).

F. Example 1

In order to demonstrate the applicability of the method, a detailed analysis of a pump is carried out. The nominal parameters of the pump are $E_*t = 13569$ Pa·m, d = 1 μ m, $\epsilon_0 = 8.85 \ 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$, $\kappa = 3$, $p_0 = 8 \ 10^5$ Pa. For example, if polyimide is used as diaphragm material, the mentioned E_*t value will approximately correspond to a diaphragm thickness of 5 μ m. The cavity geometry is defined by $z_0 = 4r^2$ with radius R = 0.005 m. It follows that the depth of the cavity is 10^{-4} m = $100 \ \mu$ m.

First, the equilibrium condition is obtained for an applied voltage V = 50 V. Fig. 5 shows the variation of $U = U_s + U_p - U_c$ with R_1 . Equilibrium is reached when $(dU/dR_1) = 0$, i.e., when $R_1 = 3.798$ mm. Fig. 6 shows cavity and diaphragm configuration at equilibrium. The corresponding pressure of the gas in the cavity is $p + p_0 = 800268$ Pa.

The dependence of pressure rise p on several parameters is studied next. Fig. 7 shows the variation of pressure with



Fig. 6. Two-dimensional axisymmetric view of the cavity and diaphragm configurations at equilibrium for pump 1. V = 50 V, R = 5 mm, $d = 1 \mu$ m, $E_*t = 13569$ Pa·m. The corresponding pressure in the cavity is 800268 Pa with $R_1 = 3.798$ mm for an initial pressure $p_0 = 800000$ Pa.



Fig. 7. Variation of p, pressure rise, in pump 1 as a function of V, applied voltage, for three different values of lumped parameter E_*t , where $d = 1 \,\mu$ m.



Fig. 8. Variation of p, pressure rise, in pump 1 as a function of V, applied voltage, for three different values of d, dielectric layer thickness, where $E_*t = 13569$ Pa·m.

applied voltage for three possible values of E_*t . Fig. 8 shows the pressure variation with voltage for three different values of dielectric thickness. Note that at the maximum voltage, p increases only by about 1 kPa, when E_*t changes from 3000 Pa·m to 20 000 Pa·m. But at the same voltage, when the dielectric thickness d decreases from 2 μ m to 0.2 μ m, p goes from about 2.5 kPa to 25 kPa. In reality, the maximum voltage that can be applied during actuation depends on the dielectric breakdown field V/d of the dielectric material between the diaphragm electrode and the cavity floor.

One may pose the question, whether the strength of the diaphragm is a limiting factor. For 50 V, the model predicts a uniform radial stress of 8.4 MPa and a uniform tangential stress of 2.5 MPa. If the applied voltage is now raised to 200 V, radial stress becomes 30.6 MPa and the tangential

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stress becomes 9.2 MPa. These stresses are very low when compared to the yield stresses of many rigid, glassy polymers. For example, many polyimides have yield stresses above 100 MPa [14]. This shows that the factor limiting the maximum attainable pressure is not the material strength but the electric field and the dielectric breakdown. On the other hand, although stresses in the diaphragm are low, they may be severe enough to initiate delamination between the different layers of the diaphragm. This may be another possible cause of failure.

One of the simplifications employed on the diaphragm deformation in the above method is $\epsilon_{\theta} = 0$. Furthermore, the radial strain ϵ_r is assumed to be uniform along the radius of the diaphragm. In order to verify these approximations, a more generalized analysis of the diaphragm is carried out where the above two restrictions are removed. The analysis is due to Hencky [13]. Note that Hencky's approach only modifies the behavior of the diaphragm, it does not address the electro-elastic coupled problem. Hence, in the following, the method developed in this paper for solving the equilibrium configuration of the pump is applied again with Hencky membrane.

G. Hencky's Diaphragm Problem

A detailed derivation of the equations of equilibrium of a plate under uniform lateral loading, where the lateral deflections are either small or large as compared to the thickness of the plate, are given by Föppl [15].

In the first case, where the lateral deflections are small, the middle plane of the plate is assumed to be the neutral surface of the plate, since its strain can be ignored as compared to the strains of other parallel planes in the membrane. In the calculations of strains of these planes as a function of their distances from the neutral surface, it is assumed that normals to the neutral surface remain linear and normal to the shape which the neutral surface takes during deformation. Then the problem takes the characteristics of a bending problem, where moments and shear stresses have to be taken into account.

The second case, where the lateral deflections of the plate are large as compared to its thickness, but still small with respect to the other dimensions, e.g., the diameter of a circular plate, is of interest to us, because the deflections of our membrane are several times higher than its thickness. Since the thickness of the diaphragm is really small, the lateral location of a material point within the diaphragm can be ignored. And due to the symmetry, tangential coordinates do not enter the solution either. So the lateral deflection z_f of a point in the diaphragm depends only on the radial coordinate of that point, when we think in terms of polar coordinates with the origin at the center of the diaphragm. Now, there are no moments created with respect to the middle plane, and there are no shear stresses along the thickness direction. There are also no in-plane shear stresses, since the polar coordinates and the principal coordinates under uniform lateral loading coincide.

Hencky [13] solves the latter problem for a circular membrane with clamped edges for the case of no residual stresses. The loading is uniform and lateral, i.e., normal to the plane of the undeformed membrane. Note that this is not true pressure. He finds a series solution to the equations of equilibrium. The following equations are Hencky's solutions, which give the radial and tangential stresses σ_r and σ_t , respectively, and film shape z_f in the region $-R_1 \leq r \leq R_1$ at equilibrium for the uniform lateral loading p

$$\sigma_{r} = \frac{1}{4} \left(Ep^{2} \frac{R_{1}^{2}}{t^{2}} \right)^{1/3} \left[b_{0} + b_{2} \left(\frac{r}{R_{1}} \right)^{2} + b_{4} \left(\frac{r}{R_{1}} \right)^{4} + \cdots \right]$$
(18)
$$\sigma_{t} = \frac{1}{4} \left(Ep^{2} \frac{R_{1}^{2}}{t^{2}} \right)^{1/3} \left[b_{0} + 3b_{2} \left(\frac{r}{R_{1}} \right)^{2} + 5b_{4} \left(\frac{r}{R_{1}} \right)^{4} + \cdots \right]$$
(19)
$$z_{f} = R_{1} \left(\frac{pR_{1}}{Et} \right)^{1/3} \left[\sum_{0}^{n} a_{n} - \left(\frac{r}{R_{1}} \right)^{2} \left(a_{0} + a_{2} \left(\frac{r}{R_{1}} \right)^{2} + a_{4} \left(\frac{r}{R_{1}} \right)^{4} + \cdots \right) \right].$$
(20)

Note that the only coordinate entering the equations is the radial one in accordance to the above discussion. There is an algebraic mistake in Hencky's original work in the calculations of the coefficients a_i and b_i of the power series in (18)–(20). The correct coefficients can be found in Fichter's work [16]. They directly depend on the Poisson's ratio. Fichter also modifies the problem of uniform lateral loading for the case of true pressure, which actually corresponds to our case, because we also have a radial component of the pressure acting on the diaphragm. Fichter concludes that both uniform lateral loading and true pressure give similar results under small loadings. Defining a dimensionless parameter $q = (pR_1)/(Et)$ at equilibrium, Fichter shows that the differences between lateral loading and true pressure cases increase with increasing q. For example, if q = 0.01, stress at r = 0, i.e., at the center, for true pressure case exceeds that for uniform lateral loading by 3.5%. Toward the edge, radial stress predicted by uniform loading exceeds the radial stress predicted by true pressure approximately by the same amount. In our simulations we take $\nu = 0.3$. Since E_*t is not an independent variable in Hencky's solution, we take E = 12.34779 GPa and $t = 1 \ \mu m$. Under these conditions, the maximum pressure attained with the Hencky membrane is 18.64 kPa at $R_1 = 2.732$ mm for the extreme case V = 250 V and $d = 0.2 \ \mu m$. This corresponds to $q \approx 0.004$. So the deviation in our case is even less than 3.5%. Thus, the results due to uniform lateral loading as given by (18)–(20) with the correct coefficients reported by Fichter are accepted as the results corresponding to true pressure case.

Fig. 9 shows the profile of the diaphragm obtained from the Hencky solution compared with the proposed shape given by (5). $(V = 50 \text{ V}, R = 5 \text{ mm}, d = 1 \mu\text{m}$. In the proposed model $E_*t = 13569 \text{ Pa} \cdot \text{m}$, which corresponds to $\nu = 0.3$, E = 12.34779 GPa and $t = 1 \mu\text{m}$ in Hencky solution as stated in the last paragraph.) The proposed shape corresponds to a pressure rise of 268 Pa, while Hencky profile yields a pressure rise of 202 Pa. The radial stress in Hencky diaphragm under these conditions varies from 6 MPa at $r = R_1$ to 8.5 MPa at the center, whereas the proposed diaphragm of this paper has a uniform radial stress of 8.4 MPa. Similarly, the tangential



Fig. 9. Comparison of diaphragm profiles of pump 1 with Hencky diaphragm and with the simplified diaphragm proposed in the paper. The conditions are the same as those of Fig. 6. Lumped parameter $E_*t = 13569$ Pa·m corresponds to E = 12.34779 GPa, $\nu = 0.3$ and t = 1 μ m in Hencky analysis. The pressure attained with the proposed diaphragm shape is 800268 Pa with $R_1 = 3.798$ mm, whereas the configuration with the Hencky shape gives a pressure of 800202 Pa with $R_1 = 3.885$ mm for an initial pressure $p_0 = 800000$ Pa.



Fig. 10. Comparison of p, pressure rise, developed in the cavity of pump 1 with Hencky diaphragm and with the simplified diaphragm proposed in this paper. The values E = 12.34779 GPa, $\nu = 0.3$ and $t = 1 \ \mu$ m are used for Hencky diaphragm. They correspond to the lumped parameter $E_* t = 13569$ Pa·m used in the proposed model. The initial pressure $p_0 = 800000$ Pa.

stress in Hencky diaphragm varies from 2 MPa at $r = R_1$ to 8.5 MPa at the center, whereas the proposed diaphragm has a uniform tangential stress of 2.5 MPa.

Finally, Fig. 10 compares the pressures developed in the cavities. The deviation between the pressures predicted by both methods remains around 30–35% within the range covered in Fig. 10. Clearly, the proposed model of the diaphragm overestimates the pressures at equilibrium, since the diaphragm with $\epsilon_{\theta} = 0$ proposed in this paper is stiffer than the Hencky diaphragm, where $\epsilon_{\theta} \neq 0$. In reality, however, the membrane has bending stiffness, which is ignored by both the Hencky diaphragm and the proposed diaphragm. Thus, the pressure will be higher than that predicted by Hencky's solution. Due to its simplicity, we will use the model of the diaphragm proposed in this paper for the analysis of pump 2.

V. ANALYSIS OF PUMP 2—ANNULAR CAVITY

A schematic of pump 2 is shown in Fig. 2 after actuation. Unlike pump 1, the diaphragm here is held to the substrate by electrostatic forces along an inner and an outer periphery of radii r_1 and r_2 , respectively. It is again under a net pressure p. The applied voltages for the two contact regions are V_1 and V_2 , respectively.

A. Membrane Deformation Under Uniform Pressure

The diaphragm profile between r_1 and r_2 is defined by $z_f(r)$. The cavity profile is given by z_0 . Consider a thin strip of film of width dr as a free body in Fig. 11. The inner and outer radii of the strip are r_a and r_b such that $dr = r_b - r_a$. The uniform tension per unit length of the perimeter is S. The equilibrium of forces along the vertical direction gives

$$2\pi r_b S \frac{dz_f}{dr} \bigg|_b - 2\pi r_a S \frac{dz_f}{dr} \bigg|_a - 2\pi r p \, dr = 0.$$
(21)

Thus

$$\frac{d}{dr}\left(\frac{rdz_f}{dr}\right) = \frac{pr}{S}.$$
(22)

The solution is given by

$$z_f = \frac{pr^2}{4S} + C_1 \ln r + C_2$$

where C_1 and C_2 are constants. They are obtained from the boundary conditions $z_f(r_1) = z_1$ and $z_f(r_2) = z_2$ (Fig. 2) so that

$$z_f = p_s r_2 (f_1 + f_2) + f_3 \tag{23}$$

where $p_s = p/S$ and

$$f_1 = \frac{r_2}{4} \left(1 - \frac{r^2}{r_2^2} \right)$$

$$f_2 = -\frac{r_2 \left(1 - r_1^2 / r_2^2 \right)}{4 \ln(r_1 / r_2)} \ln\left(\frac{r}{r_2}\right)$$

$$f_3 = z_2 + \frac{z_1 - z_2}{\ln(r_1 / r_2)} \ln\left(\frac{r}{r_2}\right).$$

Note that p_s , r_1 , and r_2 are unknowns and need to be solved to define z_f .

B. Solution for p_s

 p_s is solved using linear elasticity of the membrane and the constitutive law $(p + p_0)v = C_*$ of the gas, where p_0 is the pressure outside the cavity as well as the initial pressure (prior to actuation) in the cavity. The uniform radial strain of the membrane

$$\epsilon_r = \frac{1}{R} \left[\int_0^R \sqrt{1 + z'^2} \, dr - R \right]$$

= $\frac{1}{R} \left[\int_0^R (1 + z'^2/2) \, dr - R \right]$
= $\frac{1}{2R} \int_0^R z'^2 \, dr.$ (24)

Here z defines the shape of the film from r = 0 to r = R. In the region $r \sim (0, r_1)$, $z = z_0(r)$ (the cavity shape),



Fig. 11. Above: The free body diagram of a strip of the diaphragm for pump 2 with inner and outer radii r_a and r_b , respectively. Below: Two-dimensional axisymmetric view of the cavity and diaphragm configurations of pump 2 at equilibrium. Here $V_1 = V_2 = 200$ V, R = 5 mm, $d = 1 \mu$ m, $E_*t = 13569$ Pa·m. The corresponding pressure in the cavity is 801534 Pa with $r_1 = 0.468$ mm and $r_2 = 3.705$ mm for an initial pressure $p_0 = 800000$ Pa.

in $r \sim (r_1, r_2)$, $z = z_f(r)$ (shape of the diaphragm under pressure), and in $r \sim (r_2, R)$, $z = z_0(r)$. Thus, from (24)

$$\epsilon_r = \frac{1}{2R} \left[\int_0^{r_1} z_0^{\prime 2} dr + \int_{r_1}^{r_2} z_f^{\prime 2} dr + \int_{r_2}^R z_0^{\prime 2} dr \right].$$
(25)

The radial line tension S, can then be represented in terms of p_s using (23) and (25)

$$S = \sigma_r t = \epsilon_r E_* t$$

= $C_s p_s^2 r_2^2 - D_s p_s r_2 + E_s$ (26)

where

$$C_{s} = \frac{E_{*}t}{2R} \int_{r_{1}}^{r_{2}} (f_{1}' + f_{2}')^{2} dr,$$

$$D_{s} = \frac{E_{*}t}{R} \int_{r_{1}}^{r_{2}} (f_{1}' + f_{2}')f_{3}' dr,$$

$$E_{s} = \frac{E_{*}t}{2R} \left[\int_{0}^{r_{1}} z_{0}'^{2} dr + \int_{r_{1}}^{r_{2}} f_{3}'^{2} dr + \int_{r_{2}}^{R} z_{0}'^{2} dr \right].$$

The ideal gas law, $(p + p_0)v = p_0v_0 = C_*$, together with the volume of the compressed gas $v = \int_{r_1}^{r_2} 2\pi r(z_0 - z_f) dr$, is employed to obtain

$$p = \frac{C_*}{A_p + B_p p_s r_2} - p_0 \tag{27}$$

where

$$A_p = \int_{r_1}^{r_2} 2\pi r(z_0 - f_3) \, dr, \quad B_p = \int_{r_1}^{r_2} 2\pi r(f_1 + f_2) \, dr$$

Dividing (17) by (26) and rearranging, the equation for p_s is obtained

$$p_s(A_p + B_p p_s r_2) (C_s p_s^2 r_2^2 - D_s p_s r_2 + E_s) + p_0(A_p + B_p p_s r_2) = C_*.$$
(28)

Note that p_s may have four solutions, but the physically meaningful solution must satisfy $p_s > 0$, since p > 0, S > 0when the pump is actuated. It can be shown that *there exists* a unique solution for p_s in the domain $p_s > 0$ for all r_1/r_2 in $0 < r_1/r_2 \le 1$. p_s can be solved numerically from (28) for given values of r_1 and r_2 . The solutions for r_1 and r_2 will be obtained from the condition of equilibrium which involves the derivatives of strain, pressure, and capacitive energies, U_s , U_p and U_{\parallel} , respectively.

C. Capacitive (U_{\parallel}) , Strain (U_s) , and Pressure (U_p) Energies

 $U_{||}$: $U_{||}$ is contributed by the regions of contact between the diaphragm and the cavity. Let A_1 and A_2 denote the contact areas in $r \sim (0, r_1)$ and $r \sim (r_2, R)$, respectively (Fig. 2). Then

$$A_1 = \int_0^{r_1} 2\pi r \sqrt{1 + z_0'^2} \, dr, \quad A_2 = \int_{r_2}^R 2\pi r \sqrt{1 + z_0'^2} \, dr$$
(29)

and

$$U_{||} = \frac{1}{2} \frac{\epsilon_0 \kappa}{d} \left(A_1 V_1^2 + A_2 V_2^2 \right).$$
(30)

 U_s : The strain energy of the diaphragm is given by

$$U_s = \pi t \int_0^R \epsilon_r \sigma_r r \, dr = E_* \pi t \epsilon_r^2 R^2 / 2 \tag{31}$$

where ϵ_r is defined in (25).

 U_p : The work done on the gas U_p is given by (16). Here v_0 (the volume of the cavity before actuation) and v (volume after actuation) are given by

$$v_0 = \int_0^R 2\pi r z_0 \, dr, \quad v = \int_{r_1}^{r_2} 2\pi r (z_0 - z_f) \, dr.$$

Thus, for given voltages V_1 and V_2 , U_{\parallel} , U_s , and U_p are functions of r_1 and r_2 .

D. Equilibrium Configuration

The required values of (r_1, r_2) minimize the energy $U = U_s + U_p - U_{||}$. r_1 and r_2 are thus solved from the equilibrium condition

$$\frac{\partial U}{\partial r_i} = 0, \quad i = 1, 2. \tag{32}$$

In the following, a specific example of pump 2 is provided to demonstrate the applicability of the method developed.

E. Example 2

Fig. 11 shows the cavity of the pump defined by

$$z_0 = z_{\max} [14.64(r/R)^2 - 29.77(r/R)^3 + 15.19(r/R)^4 - 0.05(r/R)^5]$$
(33)

where $z_{\text{max}} = 112.8 \ \mu\text{m}$, so that the depth of the cavity becomes 100 μm . The relevant parameters are $d = 1 \ \mu\text{m}$, $E_*t = 13569 \text{ Pa·m}$, R = 5 mm, $\kappa = 3$, $\epsilon_0 = 8.85 \ 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$, $p_0 = 8 \ 10^5 \text{ Pa}$, and $V_1 = V_2 = 200 \text{ V}$.



Fig. 12. The variation of total energy U as a function of r_1 and r_2 . The lowest point corresponds to the equilibrium configuration shown in Fig. 11.



Fig. 13. Variation of p for pump 2 as a function of $V_1 = V_2 = V$ for three different values of lumped parameter E_*t for the case of $d = 1 \ \mu$ m. The initial pressure $p_0 = 800\ 000$ Pa.

Fig. 12 shows the variation of $U = U_s + U_p - U_{||}$ as a function of r_1 and r_2 . U is minimized when $\frac{\partial U}{\partial r_1} = 0$ and $\frac{\partial U}{\partial r_2} = 0$, which occurs at $r_1 = 0.468$ mm and $r_2 = 3.705$ mm. We obtained only one energy minimum as shown in Fig. 12 for all the results (Figs. 11–15) presented in this paper. However, we find that there are multiple minima for large values of electric field, $V/d > 400 \text{ V}/\mu\text{m}$. Thus, the nonlinear problem shows bifurcation of solution at high electric field (not treated in this paper), and such high fields should be avoided. Fig. 11 shows the equilibrium configuration of the membrane in two dimensions. In arriving at the equilibrium configuration $p_s = p/S$ were solved from (28) for each (r_1, r_2) of Fig. 12.

Figs. 13 and 14 show the dependence of pressure rise on E_*t and dielectric thickness d, respectively, as a function of applied voltage $V_1 = V_2 = V$. As in the case of single cavity pump (pump 1), the pressure increases by only 30% as E_*t increases by an order of magnitude, and at the same time it increases almost inversely with d.

Finally, the effect of cavity shape on pressure is shown in Fig. 15. Here the equilibrium pressure rise p is plotted as a function of z_{max} [(33)], which is a direct indication of the initial volume of the cavity. The figure implies that p decreases with increasing volume of the cavity. Thus, in order to attain higher pressures, one may consider a design shown in Fig. 16, where the volume of the cavity is reduced by a flat bottom.



Fig. 14. Variation of p as a function of $V_1 = V_2 = V$ for two different values of dielectric thickness d for the case of $E_*t = 13569$ Pa·m. The initial pressure $p_0 = 800000$ Pa.



Fig. 15. Variation of p as a function of z_{max} of (33), which gives a measure of the initial cavity volume of pump 2, for the case of $d = 1 \ \mu$ m and $E_*t = 13569$ Pa·m. The initial pressure $p_0 = 800\ 000$ Pa. The applied voltage in the above simulation is 200 V.

VI. CONCLUSION

This paper presents a methodology for the analysis of the electrostatic actuation of diaphragm micro pumps analytically. The method is applied to study the pressure-voltage dependence of two micro pumps, one with a simple paraboloidal cavity where the diaphragm contacts the cavity floor from the perimeter during actuation. The other has an annular cavity (a simple cavity with an island in the middle). Here the diaphragm contacts the cavity floor both from the outer and the inner perimeters. The study on both of the pumps indicates that for a given applied voltage, the pressure increases 1) almost inversely with the thickness of the dielectric between the diaphragm and the cavity floor, 2) slightly with the increase in the diaphragm thickness and elastic modulus, and 3) as the initial volume of the cavity decreases. Finally, the pressure rise in the simple cavity is higher than that in the annular cavity for similar applied voltages and overall cavity sizes. For example, pressure rise in a 100- μ m-deep and 1-cm-wide simple paraboloidal cavity along with a 5- μ m-thick polyimide diaphragm is around 6 kPa, if the dielectric layer is 1 μ m thick and the applied voltage is 250 V. This will increase to 25 kPa if the dielectric layer thickness is decreased to 0.2 μ m. A similar design with the annular cavity (1- μ m thick dielectric layer) gives rise to a pressure rise of 2 kPa instead of 6 kPa.

In these calculations, tangential strains developed in the diaphragm are ignored, i.e., $\epsilon_{\theta} = 0$. Thus the membrane is stiffer. A more detailed study for the simple cavity that includes tangential strains but ignores bending stresses showed that



Fig. 16. Cavity design with reduced volume.



Fig. 17. A design for liquid pumping. Note that there are no electrostatic fields across the liquid.

the " $\epsilon_{\theta} = 0$ " assumption leads to around 30% overestimation of the pressure rise. In reality, however, the deviation will be less, since the membrane has bending stiffness. Another simplification is that the model is based on a linear elastic material behavior for the diaphragm. A study of stresses developed in the diaphragm showed that they are well below the yield strength of many glassy polymers, e.g., polyimides, a candidate material for the diaphragm. This result also indicates that the maximum attainable pressure in the pump is limited by the dielectric breakdown rather than by the mechanical strength of the material. Furthermore, in spite of being low, stresses may be severe enough to cause delamination between the different layers of the diaphragm.

The pump designs discussed in this paper may not be applicable for liquid pumping, since the liquid may ionize when subjected to electrostatic fields. However, a minor modification in design may eliminate the restriction. For example, in the design presented in Fig. 17, the electrostatic field may be applied between the diaphragm and the cavity floor, and the diaphragm is pulled toward the floor. Thus, a cavity is formed between the diaphragm and the flat substrate, and liquid is drawn into this cavity. When the electric field is turned off, the diaphragm springs back and pumps the liquid out. Thus, the liquid is not subjected to any electric field.

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